p-Adic and Adelic Superanalysis

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Abstract

After a brief review of p-adic numbers, adeles and their functions, we consider real, p-adic and adelic superalgebras, superspaces and superanalyses. A concrete illustration is given by means of the Grassmann algebra generated by two anticommuting elements.

1 Introduction

Since Einstein's theory of (special and general) relativity, a symmetry principle has been often taken as a guidance to formulate a new and more profound physical theory. In the process of gradual discoveries more general and fundamental symmetries, a symmetry called supersymmetry (SUSY) was invented some more than three decades ago [1].

It is well known that SUSY relates basic properties between bosons and fermions. It plays very important role in construction of new fundamental models of elementary particle physics beyond the Standard Model. There are many supersymmetric field theory and supergravity models. SUSY improves situation with problem of ultraviolet divergences, and is important for point particles as well as for extended objects (strings, branes). In particular, SUSY plays a significant role in construction of String/M-theory, which is currently the best candidate for unification of all interactions and elementary constituents of matter.

Besides enormous success of SUSY, to our opinion it should be extended by the following adelic symmetry principle: a fundamental physical theory (like String/M-theory) has to be invariant under some interchange of real and p-adic number fields. For the first time, a similar principle was given by Volovich [2]. There are already some good illustrative examples of adelic symmetry in adelic

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quantum mechanics [3], [4] and adelic string product formulas [5]. To extend SUSY by adelic symmetry it is natural first to find p-adic analogs of standard SUSY (over real numbers) and then to unify results in the adelic form, which takes real and all p-adic supersymmetries simultaneously and on equal footing.

In addition to SUSY a strong motivation to consider p-adic and adelic superanalysis comes also from the quest to formulate p-adic and adelic superstring theory. A notion of p-adic string and hypothesis on the existence of non-archimedean geometry at the Planck scale were introduced by Volovich [6] and have been investigated by many researchers (reviews of an early period are in [7] and [8]). Very successful p-adic analogs of the Veneziano and Virasoro-Shapiro amplitudes were proposed in [9] as the corresponding Gel'fand-Graev [10] beta functions. Using this approach, Freund and Witten obtained [5] an attractive adelic formula $A_\infty(a,b)\prod_p A_p(a,b) = 1$, which states that the product of the crossing symmetric Veneziano (or Virasoro-Shapiro) amplitude and its all p-adic counterparts equals unity (or a definite constant). This gives possibility to consider an ordinary four-point function, which is rather complicate (a special function), as an infinite product of its inverse p-adic analogs, which are elementary functions. The ordinary crossing symmetric Veneziano amplitude can be defined by a few equivalent ways and its integral form is

$$A_\infty(a,b) = \int_{\mathbb{R}} |x|_\infty^{a-1}\,|1-x|_\infty^{b-1}\,dx, \tag{1}$$

where it is taken $\hbar = 1,\ T = 1/\pi$, and $a = -\alpha(s) = -1 - \frac{s}{2},\ b = -\alpha(t),\ c = -\alpha(u)$ with the conditions $s + t + u = -8$ and $a + b + c = 1$. According to [9] p-adic Veneziano amplitude is a simple p-adic counterpart of (1), i.e.

$$A_p(a,b) = \int_{\mathbb{Q}_p} |x|_p^{a-1}\,|1-x|_p^{b-1}\,dx, \tag{2}$$

where now $x \in \mathbb{Q}_p$. In both (1) and (2) kinematical variables a, b, c are real (or complex-valued) parameters. Thus in (2) only string world-sheet boundary x is treated as p-adic variable, and all other quantities maintain their usual real values. Unfortunately, there is a problem to extend the above product formula to the higher-point functions. Some possibilities to construct p-adic superstring amplitudes are considered in [11] (see also [12], [13], and [14]). It seems that to make further progress towards formulation of p-adic and adelic superstring theory one has previously to develop systematically the corresponding superalgebra and superanalysis.

A promising recent research in p-adic string theory has been mainly related to an extension of adelic quantum mechanics [3], [15] (see also [18]) and p-adic path integrals to string amplitudes [16] and quantum field theory [17]. Also an effective nonlinear p-adic string theory (see, e.g. [8]) with an infinite number of space and time derivatives has been recently of a great interest in the context of the tachyon condensation [19] (for a recent review, see [20]).

It is also worth mentioning successful formulation and development of p-adic and adelic quantum cosmology (see [21] and references therein) which demonstrate discreteness of minisuperspace with the Planck length ℓ_0 as the elementary one. There are also many other models in classical and quantum physics, as well as in some related fields of other sciences, which use p-adic numbers and adeles (for a recent activity, see e.g. proceedings of conferences in p-adic mathematical physics [22], [23]).

The main mathematical motivations for employment of p-adic numbers and/or adeles in modern mathematical physics are based on the following facts: (i) the field of rational numbers $\mathbb{Q}$ contains all experimental and observational numerical data; (ii) $\mathbb{Q}$ is a dense subfield not only in the field of real numbers $\mathbb{R}$ but also in all fields of p-adic numbers $\mathbb{Q}_p$; (iii) $\mathbb{R}$ and $\mathbb{Q}_p$, for all prime numbers p, exhaust all possible completions of $\mathbb{Q}$; and (iv) the local-global (Hasse-Minkowski) principle, which states that, usually, if something is valid on all local fields ($\mathbb{R}$ and $\mathbb{Q}_p$) then the same is also valid on the global field ($\mathbb{Q}$). One of the main physical motivations is related to the well-known uncertainty

$$\Delta x \geq \ell_0 = \sqrt{\frac{\hbar G}{c^3}} \approx 10^{-33} cm, \tag{3}$$

which comes from some quantum gravity considerations.Accordingly one cannot measure distances smaller than the Planck length ℓ_0. Since the derivation of (3) is based on the general assumption that real numbers and archimedean geometry are valid at all scales it means that the usual approach is broken and cannot be extended beyond the Planck scale without adequate modification which contains non-archimedean geometry. The very natural modification is to use adelic approach, since it contains real and p-adic numbers which make all possible completions of the number field $\mathbb{Q}$. As a next step it is natural to consider possible relations between adelic and supersymmetry structures.

In the sequel of this article we briefly review basic properties of p-adic numbers, adeles and their functions, and then consider p-adic and adelic superanalysis, which are important for construction and investigation of the remarkable supersymmetric models.

2 p-Adic numbers, their algebraic extensions and adeles

We review here some introductory notions on p-adic numbers, their quadratic and algebraic extensions, and adeles. For further reading one can use [24], [7], [10] and [8].

It is worth to start recalling that the first infinite set of numbers we encounter is the set $\mathbb{N}$ of natural numbers. To have a solution of the simple linear equation $x + a = b$ for any $a, b \in \mathbb{N}$, one has to extend $\mathbb{N}$ and to introduce the set $\mathbb{Z}$ of integers. Requiring that there exists solution of the linear equation $nx = m$ for any $0 \neq n, m \in \mathbb{Z}$ one obtains the set $\mathbb{Q}$ of rational numbers. Evidently these sets satisfy $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$. Algebraically $\mathbb{N}$ is a semigroup, $\mathbb{Z}$ is a ring, and $\mathbb{Q}$ is a field.

To get $\mathbb{Q}$ from $\mathbb{N}$ only algebraic operations are used, but to obtain the field $\mathbb{R}$ of real numbers from $\mathbb{Q}$ one has to employ the absolute value which is an example of the norm (valuation) on $\mathbb{Q}$. Let us recall that a norm on $\mathbb{Q}$ is a map $||\cdot|| : \mathbb{Q} \to \mathbb{R}_+ = \{x \in \mathbb{R} \,|\, x \geq 0\}$ with the following properties: (i) $||x|| = 0 \leftrightarrow x = 0$, (ii) $||x \cdot y|| = ||x|| \cdot ||y||$, and $||x + y|| \leq ||x|| + ||y||$ for all $x, y \in \mathbb{Q}$. In addition to the absolute value, for which we use usual arithmetic notation $|\cdot|_\infty$, one can introduce on $\mathbb{Q}$ a norm with respect to each prime number p. Note that, due to the factorization of integers, any rational number can be uniquely written as $x = p^\nu \, \frac{m}{n}$, where p, m, n are mutually prime and $\nu \in \mathbb{Z}$. Then by definition p-adic norm (or, in other words, p-adic absolute value) is $|x|_p = p^{-\nu}$ if $x \neq 0$ and $|0|_p = 0$. One can verify that $|\cdot|_p$ satisfies all the above conditions and moreover one has strong triangle inequality, i.e. $|x + y|_p \leq \; max\,(|x|_p, \, |y|_p)$. Thus p-adic norms belong to the class of non-archimedean (ultrametric) norms. There is only one inequivalent p-adic norm for every prime p. According to the Ostrowski theorem any nontrivial norm on $\mathbb{Q}$ is equivalent either to the $|\cdot|_\infty$ or to one of the $|\cdot|_p$. One can easily show that $|m|_p \leq 1$ for any $m \in \mathbb{Z}$ and any prime p. The p-adic norm is a measure of divisibility of the integer m by prime p: the more divisible, the p-adic smaller. By Cauchy sequences of rational numbers one can make completions of $\mathbb{Q}$ to obtain $\mathbb{R} \equiv \mathbb{Q}_\infty$ and the fields $\mathbb{Q}_p$ of p-adic numbers using norms $|\cdot|_\infty$ and $|\cdot|_p$, respectively. The cardinality of $\mathbb{Q}_p$ is the continuum as that one of $\mathbb{Q}_\infty$. p-Adic completion of $\mathbb{Z}$ gives the ring $\mathbb{Z}_p = \{x \in \mathbb{Q}_p | \; |x|_p \leq 1\}$ of p-adic integers. Denote by $\mathbb{U}_p = \{x \in \mathbb{Q}_p | \; |x|_p = 1\}$ multiplicative group of p-adic units.

Any p-adic number $0 \neq x \in \mathbb{Q}_p$ has unique representation (unlike real num-

bers) as the sum of a convergent series of the form

$$x = p^{\nu}\,(x_0 + x_1 p + x_2 p^2 + \cdots + x_n p^n + \cdots), \quad \nu \in \mathbb{Z}, \quad x_n \in \{0, 1, \cdots, p-1\}. \tag{4}$$

It resembles representation of a real number $y = \pm\, 10^{\mu}\, \sum_{k=0}^{-\infty} b_k 10^k,\ \mu \in \mathbb{Z},\ b_k \in \{0, 1, \cdots, 9\}$, but in a sense with expansion in the opposite way. If $\nu \geq 0$, then $x \in \mathbb{Z}_p$. When $\nu = 0$ and $x_0 \neq 0$ one has $x \in \mathbb{U}_p$. Any negative integer can be easily presented starting from the representation for -1:

$$-1 = p - 1\ +\ (p-1)p\ +\ (p-1)p^2 + \cdots + (p-1)p^n + \cdots\ . \tag{5}$$

Validity of (5) can be shown by elementary arithmetics, which is the same as in the real case, or treating it as the p-adic convergent geometric series.

By the analogy with the real case, one uses the norm $|\cdot|_p$ to introduce p-adic metric $d_p(x,y) = |x-y|_p$, which satisfies all necessary properties of metric with strengthened triangle inequality in the non-archimedean (ultrametric) form: $d_p(x,y)\ \leq\ max\,(\,d_p(x,z),\, d_p(z,y)\,)$. We can regard $d_p(x,y)$ as a distance between p-adic numbers x and y. Using this (ultra)metric, $\mathbb{Q}_p$ becomes ultrametric space and one can investigate the corresponding topology. Because of ultrametricity, the p-adic spaces have some exotic (from the real point of view) properties and usual illustrative examples are: a) any point of the ball $B_\mu(a) = \{x \in \mathbb{Q}_p \mid\ |x-a|_p\ \leq\ p^{\mu}\}$ can be taken as its center instead of a; b) any ball can be regarded as a closed as well as an open set; c) two balls may not have partial intersection, i.e. they are disjoint sets or one of them is a subset of the other; and c) all triangles are isosceles. $\mathbb{Q}_p$ is zerodimensional and totally disconnected topological space. $\mathbb{Z}_p$ is compact and $\mathbb{Q}_p$ is locally compact space.

$p\,\mathbb{Z}_p$ is a principal and the unique maximal ideal of $\mathbb{Z}_p$. The corresponding residue field is the quotient $\mathbb{Z}_p/p\,\mathbb{Z}_p$, which is the Galois field $\mathbb{F}_p$ with p elements.

Recall that the field $\mathbb{C}$ of complex numbers can be constructed as quadratic extension of $\mathbb{R}$ by using formal solution $x = \sqrt{-1}$ of the equation $x^2 + 1 = 0$ and denoted by $\mathbb{C} = \mathbb{R}\,(\sqrt{-1})$. All elements of $\mathbb{C}$ have the form $z = x + \sqrt{-1}\,y$ with $x, y \in \mathbb{R}$. $\mathbb{C}$ is algebraically closed, metrically complete field, and a two-dimensional vector space.

Algebraic extensions of $\mathbb{Q}_p$ also exist and have more complex structure. Quadratic extensions have the form $\mathbb{Q}_p(\sqrt{\tau})$ with elements $z = x + \sqrt{\tau}\,y$, where $x, y, \tau \in \mathbb{Q}_p$ and τ is not square element of $\mathbb{Q}_p$. For $p \neq 2$ there are three inequivalent quadratic extensions and one can take $\tau = \varepsilon,\, \varepsilon p,\, p$, where $\varepsilon = \sqrt[p-1]{1}\ \in \mathbb{Q}_p$. When $p = 2$ there are seven inequivalent quadratic extensions which may be characterized by $\tau = -1, \pm 2, \pm 3, \pm 6$. Quadratic extensions are complete but not

algebraically closed. For solution of any higher order algebraic equation one has to introduce at least one extension. Namely, the equation $x^n - p = 0$ has solution $x = \sqrt[n]{p}$ with the norm $|x|_p = p^{-\frac{1}{n}}$, which exponent is a rational and not an integer number. Algebraic closer of $\mathbb{Q}_p$, denoted by $\bar{\mathbb{Q}}_p$, is an infinite dimensional vector space over $\mathbb{Q}_p$ which is not complete. Completion of $\bar{\mathbb{Q}}_p$ gives $\mathbb{C}_p$ which is algebraically closed and metrically complete.

Real and p-adic numbers are continual extrapolations of rational numbers along all possible notrivial and inequivalent metrics. To consider real and p-adic numbers simultaneously and on equal footing one uses concept of adeles. An adele x (see, e.g. [10]) is an infinite sequence $x = (x_\infty, x_2, \cdots, x_p, \cdots)$, where $x_\infty \in \mathbb{R}$ and $x_p \in \mathbb{Q}_p$ with the restriction that for all but a finite set $\mathcal{P}$ of primes p one has $x_p \in \mathbb{Z}_p$. Componentwise addition and multiplication endow the ring structure to the set of all adeles $\mathbb{A}$, which is the union of restricted direct products in the following form:

$$\mathbb{A} = \bigcup_{\mathcal{P}} \mathbb{A}(\mathcal{P}), \quad \mathbb{A}(\mathcal{P}) = \mathbb{R} \times \prod_{p \in \mathcal{P}} \mathbb{Q}_p \times \prod_{p \notin \mathcal{P}} \mathbb{Z}_p \,. \tag{6}$$

A multiplicative group of ideles $\mathbb{I}$ is a subset of $\mathbb{A}$ with elements $x = (x_\infty, x_2, \cdots, x_p, \cdots)$, where $x_\infty \in \mathbb{R}^* = \mathbb{R} \setminus \{0\}$ and $x_p \in \mathbb{Q}_p^* = \mathbb{Q}_p \setminus \{0\}$ with the restriction that for all but a finite set $\mathcal{P}$ one has that $x_p \in \mathbb{U}_p$. Thus the whole set of ideles is

$$\mathbb{I} = \bigcup_{\mathcal{P}} \mathbb{I}(\mathcal{P}), \quad \mathbb{I}(\mathcal{P}) = \mathbb{R}^* \times \prod_{p \in \mathcal{P}} \mathbb{Q}_p^* \times \prod_{p \notin \mathcal{P}} \mathbb{U}_p \,. \tag{7}$$

A principal adele (idele) is a sequence $(x, x, \cdots, x, \cdots) \in \mathbb{A}$, where $x \in \mathbb{Q} \quad (x \in \mathbb{Q}^* = \mathbb{Q} \setminus \{0\})$. $\mathbb{Q}$ and $\mathbb{Q}^*$ are naturally embedded in $\mathbb{A}$ and $\mathbb{I}$, respectively.

Let us define an ordering on the set $\mathbb{P}$, which consists of all finite sets $\mathcal{P}_i$ of primes p, by $\mathcal{P}_1 \prec \mathcal{P}_2$ if $\mathcal{P}_1 \subset \mathcal{P}_2$. It is evident that $\mathbb{A}(\mathcal{P}_1) \subset \mathbb{A}(\mathcal{P}_2)$ when $\mathcal{P}_1 \prec \mathcal{P}_2$. Spaces $\mathbb{A}(\mathcal{P})$ have natural Tikhonov topology and adelic topology in $\mathbb{A}$ is introduced by inductive limit: $\mathbb{A} = \lim \mathrm{ind}_{\mathcal{P} \in \mathbb{P}} \mathbb{A}(\mathcal{P})$. A basis of adelic topology is a collection of open sets of the form $W(\mathcal{P}) = \mathbb{V}_\infty \times \prod_{p \in \mathcal{P}} \mathbb{V}_p \times \prod_{p \notin \mathcal{P}} \mathbb{Z}_p$, where $\mathbb{V}_\infty$ and $\mathbb{V}_p$ are open sets in $\mathbb{R}$ and $\mathbb{Q}_p$, respectively. Note that adelic topology is finer than the corresponding Tikhonov topology. A sequence of adeles $a^{(n)} \in \mathbb{A}$ converges to an adele $a \in \mathbb{A}$ if $i)$ it converges to a componentwise and $ii)$ if there exist a positive integer N and a set $\mathcal{P}$ such that $a^{(n)},\, a \in \mathbb{A}(\mathcal{P})$ when $n \geq N$. In the analogous way, these assertions hold also for idelic spaces $\mathbb{I}(\mathcal{P})$ and $\mathbb{I}$. $\mathbb{A}$ and $\mathbb{I}$ are locally compact topological spaces.

3 p-Adic and adelic analysis

$\mathbb{R}, \mathbb{Q}_p, \mathbb{C}, \mathbb{Q}_p(\sqrt{\tau}), \mathbb{C}_p, \mathbb{A}, \mathbb{I}$, and the higher p-adic algebraic extensions, form a large environment for realization of various mappings and the corresponding analyses. However only some of them have been used in modern mathematical physics. Thus, in addition to the classical real and complex analysis, the most important ones are related to the following mappings: $(i)\, \mathbb{Q}_p \;\to\; \mathbb{Q}_p\,,\; (ii)\, \mathbb{Q}_p \;\to\; \mathbb{C},\; (iii)\, \mathbb{A} \to \mathbb{A},\; (iv)\, \mathbb{A} \to \mathbb{C},\; (v)\, \mathbb{Q}_p \to \mathbb{Q}_p(\sqrt{\tau})$ and $(vi)\; \mathbb{Q}_p(\sqrt{\tau}) \;\to\; \mathbb{C}$. We will give now some information about the corresponding analyses.

$Case\,(i)\, \mathbb{Q}_p \;\to\; \mathbb{Q}_p$. All functions from the real analysis which are given by infinite power series $\sum a_n\, x^n$, where $a_n \in \mathbb{Q}$, can be regarded also as p-adic if we take $x \in \mathbb{Q}_p$. Necessary and sufficient condition for the convergence is $|a_n\, x^n|_p \to 0$ when $n \to \infty$. For example, p-adic exponential function is

$$\exp x = \sum_{n=0}^{+\infty} \frac{x^n}{n!}\,, \tag{8}$$

where the domain of convergence is $|x|_p < |2|_p$. We see that convergence is here bounded inside $p\,\mathbb{Z}_p$. Note that

$$|n!|_p = p^{-\frac{n-n'}{p-1}}\;, \tag{9}$$

where n' is the sum of digits in the expansion of n with respect to p , i.e. $n = n_0 + n_1 p + \cdots + n_k p^k$. An interesting class of functions which domain of convergence is $\mathbb{Z}_p$ has the form $F_k(x) = \sum_{n\geq 0} n! P_k(n) x^n$, where $P_k(n) = n^k + C_{k-1} n^{k-1} + \cdots + C_0$ is a polynomial in n with $C_i \in \mathbb{Z}$ (for various properties of these functions, see [25] and references therein).

Some functions can be constructed by the method of interpolation, which is based on the fact that $\mathbb{N}$ is dense in $\mathbb{Z}_p$. Using the technique of interpolation p-adic valued exponential, gamma and zeta functions are obtained.

In this case derivatives, antiderivatives and some definite integrals are well defined. However there is a problem of existence of the p-adic valued Lebesgue measure.

This kind of analysis is used in p-adic models of classical physics.

$Case\,(ii)\, \mathbb{Q}_p \;\to\; \mathbb{C}$. We deal here with complex-valued functions of a p-adic argument. Let us mention three important functions: the multiplicative character $\pi_s(x) = |x|_p^s\,,\; s \in \mathbb{C}$; the additive character $\chi_p(x) = e^{2\pi i \{x\}_p}$, where $\{x\}_p$ is

the fractional part of x , i. e. $\{x\}_p = p^{-n}(x_0 + x_1 p + \cdots + x_{n-1} p^{n-1})$; and the characteristic function on $\mathbb{Z}_p$ which is

$$\Omega_p(|x|_p) = \begin{cases} 1, & |x|_p \leq 1, \\ 0, & |x|_p > 1. \end{cases} \tag{10}$$

Many other important functions may be obtained using these three ones in some suitable ways.

Since $\mathbb{Q}_p$ and $\mathbb{Q}_p^*$ are locally compact spaces there are on them the additive dx and multiplicative d^*x Haar measures, respectively. With suitable normalization, these measures have the following properties: $d(x+a) = dx$, $a \in \mathbb{Q}_p$; $d(bx) = |b|_p \, dx$, $b \in \mathbb{Q}_p^*$; $d^*(bx) = d^*x$, $b \in \mathbb{Q}_p^*$; $d^*x = (1 - p^{-1}) \, |x|_p^{-1} \, dx$. Integration with the Haar measure is well defined. To overcome the problem with derivatives one exploits approach with p-adic pseudodifferential operators [7].

The Gel'fand-Graev p-adic gamma and beta functions are:

$$\Gamma_p(a) = \int_{\mathbb{Q}_p} \chi_p(x) \, \pi_a(x) \, |x|_p^{-1} \, dx \;=\; \frac{1 - p^{a-1}}{1 - p^{-a}} \, , \tag{11}$$

$$B_p(a,b) = \int_{\mathbb{Q}_p} \pi_a(x) \, |x|_p^{-1} \, \pi_b(1-x) \, |1-x|_p^{-1} \, dx \, , \tag{12}$$

where $a, b \in \mathbb{R}$ or $\mathbb{C}$. This beta function was used in construction of scattering amplitude (2) for p-adic open string (tachyon).

This kind of analysis is used also in p-adic quantum mechanics, quantum cosmology and quantum field theory.

$Case\,(iii)\; \mathbb{A} \;\rightarrow\; \mathbb{A}$. This case is an adelic collection of real and p-adic mappings which enables to consider simultaneously and on equal footing real and all p-adic aspects of a classical Lagrangian (and Hamiltonian) system. In such case parameters for a given system should be treated as rational numbers. Equations of motion must have an adelic solution, i. e. function and its argument must have the form of adeles.

$Case\,(iv)\; \mathbb{A} \;\rightarrow\; \mathbb{C}$. In this case functions are complex-valued while their arguments are adeles. The related analysis is used in adelic approach to quantum mechanics [3], [15], quantum cosmology [21], quantum field theory [17] and string theory [8], [14], [16]. Many important complex-valued functions from real and p-adic analysis can be easily extended to this adelic case. Adelic multiplicative and additive characters are:

$$\pi_s(x) = |x|^s = |x_\infty|_\infty^s \prod_p |x_p|_p^s \, , \quad x \in \mathbb{I} \, , \quad s \in \mathbb{C} \, , \tag{13}$$

$$\chi(x) = \chi_\infty(x_\infty) \prod_p \chi_p(x_p) = e^{-2\pi i x_\infty} \prod_p e^{2\pi i \{x_p\}_p}\,, \quad x \in \mathbb{A}\,. \tag{14}$$

Since all except finite number of factors in (13) and (14) are equal to unity, it is evident that these infinite products are convergent. One can show that $\pi_s(x) = 1$ if x is a principal idele, and $\chi(x) = 1$ if x is a principal adele, i. e.

$$|x|_\infty^s \prod_p |x|_p^s = 1\,, \quad x \in \mathbb{Q}^*\,, \quad s \in \mathbb{C}\,, \tag{15}$$

$$\chi_\infty(x) \prod_p \chi_p(x) = e^{-2\pi i x} \prod_p e^{2\pi i \{x\}_p} = 1\,, \quad x \in \mathbb{Q}\,. \tag{16}$$

It is worth noting that expressions (15) and (16) for $s = 1$ represent the simplest adelic product formulas, which clearly connect real and p-adic properties of the same rational number. In fact, the formula (15), for $s = 1$, connects usual absolute value and p-adic norms at the multiplicative group of rational numbers $\mathbb{Q}^*$.

Maps $\varphi_{\mathcal{P}} : \mathbb{A} \to \mathbb{C}$, which have the form

$$\varphi_{\mathcal{P}}(x) = \varphi_\infty(x_\infty) \prod_{p \in \mathcal{P}} \varphi_p(x_p) \prod_{p \notin P} \Omega_p(|x_p|_p)\,, \tag{17}$$

where $\varphi_\infty(x_\infty)$ are infinitely differentiable functions and fall to zero faster than any power of $|x_\infty|_\infty$ as $|x_\infty|_\infty \to \infty$, and $\varphi_p(x_p)$ are locally constant functions with compact support, are called elementary functions on $\mathbb{A}$. All finite linear combinations of the elementary functions (17) make the set $\mathcal{S}(\mathbb{A})$ of Schwartz-Bruhat functions $\varphi(x)$.

$\mathbb{A}$ is a locally compact ring and therefore there is the corresponding Haar measure, which is product of the real and all p-adic additive Haar measures. The Fourier transform of the Schwartz-Bruhat functions $\varphi(x)$ is

$$\tilde{\varphi}(\xi) = \int_{\mathbb{A}} \varphi(x)\, \chi(x\xi)\, dx \tag{18}$$

and it maps $\mathcal{S}(\mathbb{A})$ onto $\mathcal{S}(\mathbb{A})$. The Mellin transform of $\varphi(x) \in \mathcal{S}(\mathbb{A})$ is defined using the multiplicative character $|x|^s$ in the following way:

$$\Phi(s) = \int_{\mathbb{I}} \varphi(x)\, |x|^s\, d^*x = \int_{\mathbb{R}} \varphi_\infty(x_\infty)\, |x_\infty|_\infty^{s-1}\, dx_\infty \times \prod_p \int_{\mathbb{Q}_p} \varphi_p(x_p)\, |x_p|_p^{s-1} \frac{dx}{1 - p^{-1}}\,, \quad Re\, s > 1. \tag{19}$$

$\Phi(s)$ may be analytically continued on the whole complex plane, except $s = 0$ and $s = 1$, where it has simple poles with residues $-\varphi(0)$ and $\tilde{\varphi}(0)$, respectively. Denoting by $\tilde{\Phi}$ the Mellin transform of $\tilde{\varphi}$ then there is place the Tate formula [10]

$$\Phi(s) = \tilde{\Phi}(1-s)\,. \tag{20}$$

If we take $\varphi(x) = \sqrt[4]{2}\; e^{-\pi x_\infty^2}\, \prod_p \Omega_p(|x_p|_p)$, which is the simplest ground state of the adelic harmonic oscillator [15], then from the Tate formula (20) one gets the well-known functional relation for the Riemann zeta function, i. e.

$$\pi^{-\frac{s}{2}}\, \Gamma\Big(\frac{s}{2}\Big)\, \zeta(s) \;=\; \pi^{\frac{s-1}{2}}\, \Gamma\Big(\frac{1-s}{2}\Big)\, \zeta(1-s)\,. \tag{21}$$

We would like to emphasize this connection between the harmonic oscillator and the Riemann zeta function.

In the last two cases, i. e. $(v)\, \mathbb{Q}_p \;\to\; \mathbb{Q}_p(\sqrt{\tau})$ and $(vi)\; \mathbb{Q}_p(\sqrt{\tau}) \;\to\; \mathbb{C}$, p-adic quadratic extensions are used for values of functions and as values of the argument, respectively. The analysis for $(v)\, \mathbb{Q}_p \;\to\; \mathbb{Q}_p(\sqrt{\tau})$ is developed and used for a new type of non-archimedean quantum mechanics (see, monograph [28] and references therein). Complex-valued functions of arguments from $\mathbb{Q}_p(\sqrt{\tau})$ are also considered [10] and employed for construction of the Virasoro-Shapiro amplitudes for scattering of the p-adic closed strings [9]. Let us also mention that some mappings $\mathbb{Q}_p(\sqrt{\tau}) \;\to\; \mathbb{Q}_p(\sqrt{\tau})$ have been used for investigation of classical dynamical systems (see, book [29] and references therein)

4 p-Adic and adelic superalgebra and superspace

As a next step to superanalysis we are going to review here real and p-adic superalgebra and superspace along approach introduced by Vladimirov and Volovich [26], [27] and elaborated by Khrennikov [30] (see also [28]). Then, in the way initiated by Dragovich [31], we shall generalize this approach to the adelic case.

Let $L(\mathbb{Q}_v) = L_0(\mathbb{Q}_v) \,\oplus\, L_1(\mathbb{Q}_v)$ be Z_2-graded vector space over $\mathbb{Q}_v,\ (v = \infty, 2, 3, \cdots, p, \cdots)$, where elements $a \in L_0(\mathbb{Q}_v)$ and $b \in L_1(\mathbb{Q}_v)$ have even $(p(a) = 0)$ and odd $(p(b) = 1)$ parities. Thus $L_0(\mathbb{Q}_v)$ and $L_1(\mathbb{Q}_v)$ are vector subspaces of different parity. Such $L(\mathbb{Q}_v)$ space becomes v-adic (i. e. real and p-adic) superalgebra, denoted by $\Lambda(\mathbb{Q}_v) = \Lambda_0(\mathbb{Q}_v) \,\oplus\, \Lambda_1(\mathbb{Q}_v)$, if it is endowed by an associative algebra with unity and multiplication with parity defined by $p(ab) \equiv p(a) + p(b)\, (mod\, 2)$. Product of two elements of the same (different) parity has even (add) parity.

Supercommutator is defined in the usual way: $[a,b\} = a\,b - (-1)^{p(a)p(b)} b\,a$. Superalgebra $\Lambda(\mathbb{Q}_v)$ is called (super)commutative if $[a,b\} = 0$ for any $a\,,b$ which are elements of $\Lambda_0(\mathbb{Q}_v)$ and $\Lambda_1(\mathbb{Q}_v)$.

To obtain a Banach space from the commutative superalgebra (CSA) one has to introduce the corresponding norm

$$||fg||_v \leq ||f||_v\, ||g||_v\,, \quad f,g \in \Lambda(\mathbb{Q}_v), \tag{22}$$

which is at the end related to the absolute value $|\cdot|_\infty$ for the real case and to p-adic norms $|\cdot|_p$ for p-adic cases.

As illustrative examples of commutative superalgebras one can consider finite dimensional v-adic Grassmann algebras $G(\mathbb{Q}_v : \eta_1, \eta_2, \cdots, \eta_k)$ which dimension is 2^k and generators $\eta_1, \eta_2, \cdots, \eta_k$ satisfy anticommutative relations $\eta_i\eta_j + \eta_j\eta_i = 0$. These $\eta_i\,\eta_j$ can be realized as: 1) product of annihilation operators $a_i\,a_j$ for fermions, 2) exterior product $dx^i \wedge dx^j$, and 3) as product of some matrices. One can write

$$G(\mathbb{Q}_v : \eta_1, \cdots, \eta_k) = G_0(\mathbb{Q}_v : \eta_1, \cdots, \eta_k) + G_1(\mathbb{Q}_v : \eta_1, \cdots, \eta_k), \tag{23}$$

where $G_0(\mathbb{Q}_v : \eta_1, \eta_2, \cdots, \eta_k)$ and $G_1(\mathbb{Q}_v : \eta_1, \eta_2, \cdots, \eta_k)$ contain sums of 2^{k-1} terms with even and add number of algebra generators η_i, respectively. Note that the role of commuting and anticommuting coordinates will play these sums with even and add parity and not the coefficients in expansion over products of η_i. As CSA one can also use the infinite dimensional Grassmann algebra.

Let $\Lambda(\mathbb{Q}_v)$ be a fixed commutative v-adic superalgebra. v-Adic superspace of dimension (n,m) over $\Lambda(\mathbb{Q}_v)$ is

$$\mathbb{Q}^{n,m}_{\Lambda(\mathbb{Q}_v)} = \Lambda^n_0(\mathbb{Q}_v) \times \Lambda^m_1(\mathbb{Q}_v)\,, \tag{24}$$

where

$$\Lambda^n_0(\mathbb{Q}_v) = \underbrace{\Lambda_0(\mathbb{Q}_v) \times \cdots \times \Lambda_0(\mathbb{Q}_v)}_{n}, \quad \Lambda^m_1(\mathbb{Q}_v) = \underbrace{\Lambda_1(\mathbb{Q}_v) \times \cdots \times \Lambda_1(\mathbb{Q}_v)}_{m}\,. \tag{25}$$

This superspace is an extension of the standard v-adic space, which has now n commuting and m anticommuting coordinates.

The points of the superspace $\mathbb{Q}^{n,m}_{\Lambda(\mathbb{Q}_v)}$ are

$$\begin{aligned} X^{(v)} &= (X^{(v)}_1, X^{(v)}_2, \cdots, X^{(v)}_n, X^{(v)}_{n+1}, \cdots, X^{(v)}_{n+m}) \\ &= (x^{(v)}_1, x^{(v)}_2, \cdots, x^{(v)}_n, \theta^{(v)}_1, \cdots, \theta^{(v)}_m) = (x^{(v)}, \theta^{(v)}), \end{aligned} \tag{26}$$

where coordinates $x_1^{(v)}, x_2^{(v)}, \cdots, x_n^{(v)}$ are commuting, with $p(x_i^{(v)}) = 0$, and $\theta_1^{(v)}$, $\theta_2^{(v)}, \cdots, \theta_m^{(v)}$ are anticommuting (Grassmann) ones, with $p(\theta_j^{(v)}) = 1$. Since the supercommutator $[X_i^{(v)}, X_j^{(v)}\} = X_i^{(v)} X_j^{(v)} - (-1)^{p(X_i^{(v)})p(X_j^{(v)})} X_j^{(v)} X_i^{(v)} = 0$, the coordinates $X_i^{(v)},\ (i = 1, 2, \cdots, n+m)$ are called supercommuting.

A norm of $X^{(v)}$ can be defined as

$$||X^{(v)}||_v = \begin{cases} \sum_{i=1}^n ||x_i^{(\infty)}||_\infty + \sum_{j=1}^m ||\theta_j^{(\infty)}||_\infty, & v = \infty, \\ \max_{1\leq i \leq n,\, 1 \leq j \leq m} (\, ||x_i^{(p)}||_p \,, ||\theta_j^{(p)}||_p \,), & v = p\,, \end{cases} \tag{27}$$

where $||X^{(p)}||_p\,,\ ||x_i^{(p)}||_p$ and $||\theta_j^{(p)}||_p$ are non-archimedean norms. In the sequel, to decrease number of indices we often omit some of them when they are understood from the context.

We can now turn to the adelic case of superalgebra and superspace. Let us start with the corresponding Z_2-graded vector space over $\mathbb{A}$ as

$$L(\mathbb{A}) = \bigcup_{\mathcal{P}} L(\mathcal{P})\,, \quad L(\mathcal{P}) = L(\mathbb{R}) \times \prod_{p \in \mathcal{P}} L(\mathbb{Q}_p) \times \prod_{p \notin \mathcal{P}} L(\mathbb{Z}_p)\,, \tag{28}$$

where $L(\mathbb{Z}_p) = L_0(\mathbb{Z}_p) \oplus L_1(\mathbb{Z}_p)$ is a graded space over the ring of p-adic integers $\mathbb{Z}_p$ (and $\mathcal{P}$ is a finite set of primes p). Graded vector space (28) becomes adelic superalgebra

$$\Lambda(\mathbb{A}) = \bigcup_{\mathcal{P}} \Lambda(\mathcal{P})\,, \quad \Lambda(\mathcal{P}) = \Lambda(\mathbb{R}) \times \prod_{p \in \mathcal{P}} \Lambda(\mathbb{Q}_p) \times \prod_{p \notin \mathcal{P}} \Lambda(\mathbb{Z}_p)\,, \tag{29}$$

by requiring that $\Lambda(\mathbb{R})\,, \Lambda(\mathbb{Q}_p)\,,$ and $\Lambda(\mathbb{Z}_p) = \Lambda_0(\mathbb{Z}_p) \oplus \Lambda_1(\mathbb{Z}_p)$ are superalgebras. Adelic supercommutator may be regarded as a collection of real and all p-adic supercommutators. Thus adelic superalgebra (29) is commutative. An example of commutative adelic superalgebra is the following adelic Grassmann algebra:

$$G(\mathbb{A} : \eta_1, \eta_2, \cdots, \eta_k) = \bigcup_{\mathcal{P}} G(\mathcal{P} : \eta_1, \eta_2, \cdots, \eta_k) \tag{30}$$

$$\begin{gathered} G(\mathcal{P} : \eta_1, \cdots, \eta_k) = G(\mathbb{R} : \eta_1, \cdots, \eta_k) \\ \times \prod_{p \in \mathcal{P}} G(\mathbb{Q}_p : \eta_1, \cdots, \eta_k) \times \prod_{p \notin \mathcal{P}} G(\mathbb{Z}_p : \eta_1, \cdots, \eta_k). \end{gathered} \tag{31}$$

Banach commutative superalgebra for $\Lambda(\mathcal{P})$ defined in (29) obtains by taking all of $\Lambda(\mathbb{Q}_\infty)$, $\Lambda(\mathbb{Q}_p)$, $\Lambda(\mathbb{Z}_p)$ to be Banach spaces. The $\Lambda(\mathcal{P})$ will have the corresponding Tikhonov topology. Then the corresponding Banach adelic space is

inductive limit $\Lambda(\mathbb{A}) = \lim \mathrm{ind}_{\mathcal{P}\in\mathbb{P}}\, \Lambda(\mathcal{P})$, and in this way it gets an adelic topology.

Adelic superspace of dimension (n,m) has the form

$$\mathbb{A}^{n,m}_{\Lambda(\mathbb{A})} = \bigcup_{\mathcal{P}} \mathbb{A}^{n,m}_{\Lambda(\mathbb{A})}\left(\mathcal{P}\right), \quad \mathbb{A}^{n,m}_{\Lambda(\mathbb{A})}\left(\mathcal{P}\right) = \mathbb{R}^{n,m}_{\Lambda(\mathbb{R})} \times \prod_{p\in\mathcal{P}} \mathbb{Q}^{n,m}_{\Lambda(\mathbb{Q}_p)} \times \prod_{p\notin\mathcal{P}} \mathbb{Z}^{n,m}_{\Lambda(\mathbb{Z}_p)}, \quad (32)$$

where $\mathbb{Z}^{n,m}_{\Lambda(\mathbb{Z}_p)}$ is (n,m)-dimensional p-adic superspace over Banach commutative superalgebra $\Lambda(\mathbb{Z}_p)$.

Points X of adelic superspace have the coordinate form $X = (X^{(\infty)}, X^{(2)}, \cdots, X^{(p)}, \cdots,)$, where for all but a finite set of primes $\mathcal{P}$ it has to be $||X^{(p)}||_p = \max_{1\le i\le n,\, 1\le j\le m} \left(||x_i^{(p)}||_p , ||\theta_j^{(p)}||_p \right) \le 1$, i. e. $x_i^{(p)} \in \Lambda_0(\mathbb{Z}_p)$ and $\theta_j^{(p)} \in \Lambda_1(\mathbb{Z}_p)$.

5 Elements of p-adic and adelic superanalysis

Superanalysis is related to a map from one superspace to the other. Since we have formulated here many superspaces which are distinctly valued, therefore one can introduce many kinds of mappings between them and one can get plenty of superanalyses. For instance, one can consider the following cases: real $\longrightarrow$ real , p-adic $\longrightarrow$ p-adic , adelic $\longrightarrow$ adelic, real $\longrightarrow$ complex , p-adic $\longrightarrow$ complex , and adelic $\longrightarrow$ complex . In the sequel we will restrict our consideration to the cases without complex-valued functions. In fact we will investigate two types of maps:

$$F_v : V_v \to V'_v, \qquad \Phi_{\mathbb{A}} : W(\mathcal{P}) \to W'(\mathcal{P}'), \quad (33)$$

where $V_v \subset \mathbb{Q}^{n,m}_{\Lambda(\mathbb{Q}_v)}$, $V'_v \subset \mathbb{Q}^{n,m}_{\Lambda(\mathbb{Q}_v)}$, and $W(\mathcal{P}) \subset \mathbb{A}^{n,m}_{\Lambda(\mathbb{A})}(\mathcal{P})$, $W'(\mathcal{P}') \subset \mathbb{A}^{n,m}_{\Lambda(\mathbb{A})}(\mathcal{P}')$.

Case F_v. The function $F_v(X)$ is continuous in the point $X \in V_v$ if

$$\lim_{||h||_v\to 0} ||F_v(X+h) - F_v||_v = 0\,, \quad (34)$$

and it is continuous in V_v if (34) is satisfied for all $X \in V_v$. This function F_v is superdifferentiable in $X \in V_v$ if it can be presented as

$$F_v(X+h) = F_v(X) + \sum_{i=1}^{n+m} f_i(X)\, h_i + g(X,h)\,, \quad (35)$$

where $f_i(X) \in V_v'$ and $||g(X,h)||_v \, ||h||_v^{-1} \to 0$ when $||h||_v \to 0$. Then $f_i(X)$ are called partial derivatives of F_v in the point X with respect to X_i and denoted by

$$f_i(X) = \frac{\partial F_v(X)}{\partial X_i} = \frac{\partial F_v(x,\theta)}{\partial x_i}\,, \quad f_{n+j}(X) = \frac{\partial F_v(X)}{\partial X_{n+j}} = \frac{\partial F_v(x,\theta)}{\partial \theta_j}\,, \tag{36}$$

where $i = 1, 2, \cdots, n$ and $j = 1, 2, \cdots, m$. The superdifferential is

$$DF_v(X) = \sum_{i=1}^{n+m} \frac{\partial F_v(X)}{\partial X_i}\, h_i. \tag{37}$$

If F_v is an $(n+m)$-component function then partial derivatives form $(n+m) \times (n+m)$ Jacobi matrix. The above introduced derivatives are known as the right ones. One can also introduce the left derivatives by change $f_i(X)\, h_i \to h_i\, f_i(X)$ in (35). Higher order derivatives can be introduced in the analogous way. Note that partial derivatives with odd coordinates anticommute: $\frac{\partial}{\partial\theta\, i} \frac{\partial}{\partial\theta\, j} = -\frac{\partial}{\partial\theta\, j} \frac{\partial}{\partial\theta\, i}$. Since in this approach coordinates x_i and θ_j are composed of coefficients in commutative superalgebra $\Lambda(\mathbb{Q}_v)$ there exist the corresponding Cauchy-Riemann conditions (for details, see, [26]).

Note that superdifferentiability is closely related to the Frechèt differentiability in the Banach spaces.

The corresponding integral calculus is based on appropriately constructed differential forms [27]. Integration over noncommuting variables employs the standard rules

$$\int d\theta = 0\,, \quad \int \theta\, d\theta = 1\,. \tag{38}$$

When one of the commuting coordinates x_i is the time and the others are spatial in the superspace $\mathbb{Q}^{n,m}_{\Lambda(\mathbb{Q}_v)}$, the functions are called superfields in supersymmetric physical models.

Various aspects of p-adic superanalysis have been considered in detail and many of them can be found in the papers [32], [33], [34], [35], [36].

The corresponding adelic valued functions (superfields) $\Phi_{\mathbb{A}}$ must satisfy adelic structure, i.e. $\Phi_{\mathbb{A}}(X) = (F_\infty\,(X^{(\infty)}),\; F_2\,(X^{(2)}),\; \cdots,\; F_p\,(X^{(p)}), \cdots)$ with condition $||F_p||_p \;\leq\; 1$ for all but a finite set of primes $\mathcal{P}$. According to this adelic property and the above v-adic superanalysis one obtains the corresponding adelic superanalysis.

6 Simple model of superanalysis with Grassmann algebra

According to the above approach, essential properties of superanalysis should not depend on the concrete choice of a Z_2-graded commutative Banach algebra. Hence, to realize a simple and instructive model of superanalysis it is natural to make construction over the Grassmann algebra with two anticommuting elements (generators).

Let η_1, η_2 be two fixed generators of the v-adic Grassmann algebra $G(\mathbb{Q}_v : \eta_1, \eta_2)$, i. e. η_i satisfy $\eta_1\eta_2 = -\eta_2\eta_1$ and $\eta_1\eta_1 = \eta_2\eta_2 = 0$. Then $G(\mathbb{Q}_v : \eta_1, \eta_2)$ can be presented as (super)commutative superalgebra

$$G(\mathbb{Q}_v : \eta_1, \eta_2) = \Lambda(\mathbb{Q}_v) = \Lambda_0(\mathbb{Q}_v) \oplus \Lambda_1(\mathbb{Q}_v). \tag{39}$$

The corresponding elements $x \in \Lambda_0(\mathbb{Q}_v)$ and $\theta \in \Lambda_1(\mathbb{Q}_v)$ have the form

$$x = u + v\,\eta_1\,\eta_2\,, \quad \theta = \alpha\,\eta_1 + \beta\,\eta_2\,, \tag{40}$$

where $u,\ v,\ \alpha,\ \beta \in \mathbb{Q}_v$. According to the above approach to superanalysis (see [26] and [27]) these x and θ become commuting and anticommuting v-adic variables (coordinates), respectively. One can easily verify that any two values $\theta_1 = \alpha_1\,\eta_1 + \beta_1\,\eta_2$ and $\theta_2 = \alpha_2\,\eta_1 + \beta_2\,\eta_2$ are anticommuting, i.e. $\theta_i\,\theta_j + \theta_j\,\theta_i = 0, \quad i,j = 1,2$. Since the parity $p(\eta_1) = p(\eta_2) = 1$ and $p(u_i) = p(v_i) = p(\alpha_i) = p(\beta_i) = 1$ it follows that $p(x_1) = p(x_2) = 0$ and $p(\theta_1) = p(\theta_2) = 1$.

The corresponding real and p-adic norms of x and θ in (40) are as follows:

$$||x||_v = \begin{cases} |u|_\infty + |v|_\infty\,, & v = \infty \\ \max\{|u|_p\,, |v|_p\}\,, & v = p\,, \end{cases} \tag{41}$$

$$||\theta||_v = \begin{cases} |\alpha|_\infty + |\beta|_\infty\,, & v = \infty \\ \max\{|\alpha|_p\,, |\beta|_p\}\,, & v = p\,, \end{cases} \tag{42}$$

where $|\cdot|_\infty$ and $|\cdot|_p$ are usual norms of real and p-adic numbers. With these norms (41) and (42), $\Lambda(\mathbb{Q}_\infty)$ and $\Lambda(\mathbb{Q}_p)$ become Banach superalgebras.

Now one can introduce superspace (24) with n commuting and m anticommuting coordinates

$$x_i = u_i + v_i\,\eta_1\,\eta_2\,, \quad \theta_j = \alpha_j\,\eta_1 + \beta_j\,\eta_2\,, \quad i = 1, \cdots, n\,,\ j = 1, \cdots, m\,. \tag{43}$$

According to (27), (41) and (42) the norm of v-adic superspace points $X^{(v)} = (x^{(v)}\,, \theta^{(v)})$ is

$$||X^{(v)}||_v = \begin{cases} \sum_{i=1}^{n} (|u_i|_\infty + |v_i|_\infty) + \sum_{j=1}^{m} (|\alpha_j|_\infty + |\beta_j|_\infty)\,, & v = \infty \\ \max_{i,j}\ \{\, |u_i|_p\,, |v_i|_p\,, |\alpha_j|_p\,, |\beta_j|_p \,\}\,, & v = p\,, \end{cases} \tag{44}$$

In construction of adelic superspace (32) one has to take care that $\max_{i,j}\ \{\, |u_i|_p\,, |v_i|_p\,, |\alpha_j|_p\,, |\beta_j|_p \,\} \leq 1$ for all possibilities of finite set $\mathcal{P}$.

7 Conclusion

In this article we have given a brief review of real and p-adic analysis and superanalysis on the Banach commutative superalgebra. This is approach which generalizes analysis of complex functions with many complex variables. An introduction to adelic superanalysis is also presented. Some elements of this approach are illustrated using the Grassmann algebra of two anticommuting generators. As a next step we plan to consider p-adic and adelic superanalysis with complex-valued superfields, as well as to develop adelic theory of supersymmetry and to construct p-adic analogs and adelic models of superstring and M-theory.

Acknowledgments

One of the authors (B.D.) would like to thank Prof. V. Dobrev for invitation to participate in QTS-4 and give a plenary talk on 'VI Workshop on Lie Theory and Its Applications in Physics', Varna 2005, as well as for the hospitality and stimulative scientific atmosphere. Work on this article was partially supported by the Serbian Ministry of Science and Environmental Protection under the project no. 1426, and Grants: "EU-network: quantum probability and applications" and "Swedish Royal Academy of Science for collaboration with states of former Soviet Union".

References

[1] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton Univ. Press, Princeton, 1983.

[2] I. V. Volovich, *Number Theory as the Ultimate Physical Theory*, CERN preprint, CERN-Th. 4781/87.

[3] B. Dragovich, Adelic Model of Harmonic Oscillator, *Theor. Math. Phys.* **101**, 1404-1412 (1994) ; hep-th/0402193.

[4] G.S. Djordjević, B. Dragovich and L. Nešić, Adelic Path Integrals for Quadratic Lagrangians, *Inf. Dim. Analys. Quan. Probab. and Rel. Topics* **6**, 179-195 (2003); hep-th/0105030.

[5] P.G.O. Freund and E. Witten, Adelic string amplitudes, *Phys. Lett.* **B 199**, 191-194 (1987) .

[6] I.V. Volovich, p-Adic string, *Class. Quantum Grav.* **4**, L83-L87 (1987) .

[7] V.S. Vladimirov, I.V. Volovich and E.I. Zelenov, *p-Adic Analysis and Mathematical Physics*, World Scientific, Singapore, 1994.

[8] L. Brekke and P.G.O. Freund, p-Adic Numbers in Physics, *Phys. Rep.* **233**, 1-63 (1993).

[9] P.G.O. Freund and M. Olson, Non-Archimedean strings, *Phys. Lett.* **B 199**, 186-190 (1987).

[10] I.M. Gel'fand, M.I. Graev and I.I. Piatetskii-Shapiro, *Representation Theory and Automorphic Functions* (in Russian), Nauka, Moscow, 1966.

[11] I.Ya. Aref'eva, B.G. Dragovich and I.V. Volovich, p-Adic Superstring, *Phys. Lett.* **B 214**, 339-349 (1988).

[12] L. Brekke, P.G.O. Freund, M. Olson and E. Witten, Non-archimedean string dynamics, *Nucl. Phys.* **B 302**, 365-402 (1988).

[13] Ph. Ruelle, E. Thiran, D. Werstegen and J. Weyers, Adelic String and Superstring Amplitudes, *Mod. Phys. Lett.* **A 4**, 1745-1752 (1989).

[14] V.S. Vladimirov, Adelic Formulas for Gamma and Beta Functions of One-Class Quadratic Fields: Applications to 4-Particle Scattering String Amplitudes, *Proc. Steklov Math. Institute* **228**, 67-80 (2000).

[15] B. Dragovich, Adelic Harmonic Oscillator, *Int. J. Mod. Phys.* **A 10**, 2349-2365 (1995) ; hep-th/0404160.

[16] B. Dragovich, On Adelic Strings, hep-th/0005200;

[17] B. Dragovich, On p-Adic and Adelic Generalization of Quantum Field Theory, *Nucl. Phys. B (Proc. Suppl.)* **102**, **103**, 150-155 (2001).

[18] B. Dragovich, p-Adic and Adelic Quantum Mechanics, Proc. V.A. Steklov Inst. Math. **245**, 72-85, (2004); hep-th/0312046.

[19] D. Ghoshal and A. Sen, Tachyon Condesation and Brane Descent Relations in p-Adic String Theory, *Nucl. Phys.* **B 584**, 300-312 (2000).

[20] P.G.O. Freund, *p-adic Strings and their Applications*, hep-th/0510192. To be published in [23].

[21] G.S. Djordjević, B. Dragovich, Lj. Nešić and I.V. Volovich, p-Adic and Adelic Minisuperspace Quantum Cosmology, *Int. J. Mod. Phys.* **A 17**, 1413-1433 (2002) ; gr-qc/0105050.

[22] Selected Topics of p-Adic Mathematical Physics and Analysis, *Proc. V.A. Steklov Inst. Math.* **245**, 1-303 (2004).

[23] Proceedings of the *2nd International Conference on p-adic Mathematical Physics*, Belgrade, 15-21.09 2005. To be published by American Institute of Physics (2006).

[24] W.H. Schikhof, *Ultrametric Calculus : an introduction to p-adic analysis*, Cambridge U.P., Cambridge, 1984.

[25] B. Dragovich, On Some p-adic Series with Factorials, *Lecture Notes in Pure and Applied Mathematics* **192**, 91-105 (1997).

[26] V.S. Vladimirov and I.V. Volovich, SUPERANALYSIS. I. Differential Calculus , *Teor. Mat. Fizika* **59**, 3-27 (1984).

[27] V.S. Vladimirov and I.V. Volovich, SUPERANALYSIS. II. Integral Calculus, *Teor. Mat. Fizika* **60**, 169-198 (1984).

[28] A. Khrennikov, *p-Adic Valued Distributions in Mathematical Physics*, Kluwer Academic Publisher, Dordrecht, 1994.

[29] A. Yu. Khrennikov and M. Nilsson, *p-Adic Deterministic and Random Dynamics*, Kluwer Academic Publisher, Dordrecht, 2004.

[30] A.Yu. Khrennikov, *Superanalysis* (in Russian), Nauka,Fizmatlit, Moscow, 1997 ; English translation: Kluwer Academic Publisher, Dordrecht, 1999.

[31] B. Dragovich, *Some p-Adic Aspects of Superanalysis*, in Proc. of Int. Workshop 'Supersymmetries and Quantum Symmetries' (SQS'03), Dubna, 181-186 (2004), Eds. E. Ivanov and A. Pashnev; hep-th/0401044.

[32] A.Yu. Khrennikov, Analysis on the p-adic superspace,1.Generalized functions :Gaussian distribution, *J. Math. Phys.* **33**, 1636-1642 (1992).

[33] A.Yu. Khrennikov, Analysis on the p-adic superspace,2. Differential equations, *J. Math. Phys.* **33**, 1643-1647 (1992).

[34] R. Cianci and A.Yu. Khrennikov, p-adic superanalysis, 1. Infinitely generated Banach superalgebras and algebraic groups, *J. Math. Phys.* **34**, 1986-1994 (1993).

[35] R. Cianci and A.Yu. Khrennikov, p-adic superanalysis, 2. Supermanifolds and differential operators, *J. Math. Phys.*, **34**, 1995–2003 (1993).

[36] R. Cianci and A.Yu. Khrennikov, p-adic superanalysis, 3. Super-Lie groups, *J. Math. Phys.* **35**, 1252-1269 (1994).